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Integration as Area
AP Readiness Session 6 - February

Answers to examples posted on my website

| **General Problem Steps** | **Examples** |
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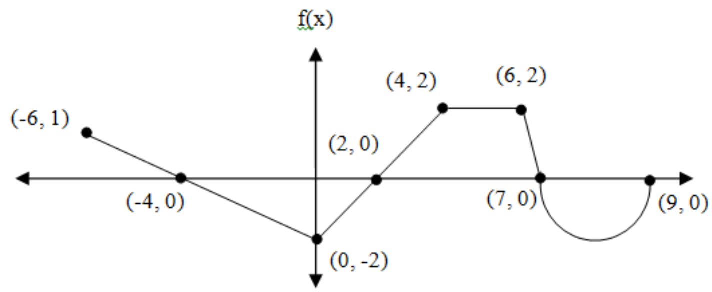
**General Problem Steps**

Given the graph of a function, find the area under the function over a given interval. (same as the definite integral of the function over the interval)
(must be linear or circular to use geometry)

- Divide up the given interval into geometric shapes
- Area below the x-axis is considered negative
- If limits of integration are left to right (low to high), then each area changes sign
- If evaluating a function defined as the integral of the given function from x to a, then you must add the initial condition given at F(a)

**Examples**




1. Find $\int_{0}^{2} f(x)dx$ = -2
2. Find $\int_{0}^{9} f(x)dx$ = $5-\frac{\pi}{2}$
3. Find $\int_{2}^{-6} f(x)dx$ = 5
4. Find $\int_{8}^{6} f(x)dx$ = $-1+\frac{\pi}{4}$

Given the graph of $f'(x)$ and $f(0)=3$

5. Find $f(-6)$ = 6
6. Find $f(6)$ = 7

**General Problem Steps**

Given a function, find the area under the graph over a given interval

- Evaluate the definite integral of the function over the interval.

**Examples**

7. Find the area under the curve $f(x)=-x^2+6x-3$ on the interval (1, 2).

Area = $\int_{1}^{2}(-x^2+6x-3)dx$ = $\left(\frac{-x^3}{3}+\frac{6x^2}{2}-3x\right)\Big|_{1}^{2}$

$$=\left(\frac{-(2)^3}{3}+\frac{6(2)^2}{2}-3(2)\right)-\left(\frac{-(1)^3}{3}+\frac{6(1)^2}{2}-3(1)\right)$$

$$=\left(\frac{-8}{3}+12-6\right)-\left(\frac{-1}{3}+3-3\right)=\frac{-8}{3}+6+\frac{1}{3}=\frac{11}{3}$$

**General Problem Steps**

Given a region defined by two functions, $f(x)$ and $g(x)$, find the area of the region.

- If $f(x)\geq g(x)$, then Area = $\int_{a}^{b}(f(x)-g(x))dx$
- If the interval from *b* to *a* is not given, then *b* and *a* are the points of intersection of $f(x)$ and $g(x)$
- Find points of intersection by setting $f(x)$ = $g(x)$ and solving for x

**Examples**

8. Find the area enclosed by $f(x)=5-x^2$ and $g(x)=x-7$

Find points of intersection: $5-x^2=x-7$

$x^2+x-12=0$, $(x+4)(x-3)=0$, $x=-4,3$

Since $5-x^2\geq x-7$ on $(-4,3)$ the area is equal to

$$\int_{-4}^{3}((5-x^2)-(x-7))dx \;=57.167$$

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|---|---|
| Given a region defined by two functions, $f(x)$ and $g(x)$, with more than two points of intersection, find the area of the region.<br><br>• Find points of intersection by setting $f(x) = g(x)$ and solving for x<br>• Determine which function is greater over each interval<br>• Split into two integrals<br><br>A = $\int_c^b (f(x)-g(x))dx + \int_a^c (g(x)-f(x))dx$<br>Where *a*, *b*, and *c* are points of intersection, *c* is between *a* and *b*, $f(x) \geq g(x)$ between *b* and *c*, and $g(x) \geq f(x)$ between *c* and *a* | 9. Find but do not evaluate an integral to represent the area enclosed by $f(x) = x^3 - 2x^2$ and $g(x) = 2x^2 - 3x$<br><br>Find points of intersection: $x^3 - 2x^2 = 2x^2 - 3x$<br>$x^3 - 4x^2 + 3x = 0$, $x(x-3)(x-1) = 0$<br>$x = 0,1,3$<br>Between 0 and 1: $f(x) \geq g(x)$ (test a value or see graph)<br>Between 1 and 3: $g(x) \geq f(x)$<br>Area =<br>$\int_1^3 ((2x^2-3x)-(x^3-2x^2))dx + \int_0^1 ((x^3-2x^2)-(2x^2-3x))dx$ |
| Given a region defined by two relations where x = some expression of y, the area can be determined by an integral in the y-direction.<br><br>Area = $\int_c^d (f(y)-g(y))dy$<br>Where c and d are the y-coordinates of the points of intersection and $f(y) \geq g(y)$ (graph of $f(y)$ is to the right of $g(y)$ ) | 10. Find but do not evaluate an integral to represent the area enclosed by the graphs of $x = 3 - y^2$ and $x = y + 1$<br><br>Find points of intersection: $3 - y^2 = y + 1$<br>$y^2 + y - 2 = 0$, $(y+2)(y-1) = 0$, $y = -2,1$<br><br>Since $3 - y^2 \geq y + 1$ on y-interval $(-2,1)$<br>Area = $\int_{-2}^1 ((3-y^2)-(y+1))dy$ |
| Given a region defined by multiple boundaries, find the area.<br><br>• Determine all intersecting points of the boundaries of the region<br>• If necessary, split the region into multiple integrals | 11. Find but do not evaluate an integral to represent the area enclosed by:<br>$y = 2\sqrt{x-1} - 3$, $y = -2x + 11$, and the x-axis<br>a. Intersection of $y = 2\sqrt{x-1} - 3$ and $y = -2x + 11$<br>$2\sqrt{x-1} - 3 = -2x + 11$, $2\sqrt{x-1} = -2x + 14$<br>$\sqrt{x-1} = -x + 7$, $x - 1 = x^2 - 14x + 49$<br>$x^2 - 15x + 50 = 0$, $(x-5)(x-10) = 0$<br>$x = 5$ ( $x = 10$ is extraneous)<br>b. Intersection of $y = 2\sqrt{x-1} - 3$ and the x-axis<br>$0 = 2\sqrt{x-1} - 3$, $3 = 2\sqrt{x-1}$, $\frac{3}{2} = \sqrt{x-1}$, $\frac{9}{4} = x - 1$<br>$\frac{13}{4} = 3.25 = x$<br>c. Intersection of $y = -2x + 11$ and the x-axis<br>$0 = -2x + 11$, $\frac{11}{2} = 5.5 = x$<br><br>Area = $\int_5^{5.5} (-2x+11)dx + \int_{3.25}^5 (2\sqrt{x-1}-3)dx$ |